

# A Curved Four-Conductor Assembly with Larger Uniform Region

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**Summary**—Most atomic physics setups require a uniform static magnetic field to define a quantization axis. The straight four-conductor assembly has been proven to be a useful structure for producing such a uniform static magnetic field. Here, we present an improved four-conductor assembly, where each conductor consists of one straight section and two curved sections. This hybrid structure arrangement offers more degrees of freedom than the conventional straight four-conductor assembly, enabling a larger uniform field region and better field spatial uniformity. The modified magnetic field generation structure can be used in many fields of atomic precision spectroscopy.

**Keywords**—static magnetic field; four-conductor; uniformity; curved section; modified four-conductor

## I. INTRODUCTION

The splitting of atomic energy levels by a static magnetic field is widely used in atomic frequency standards and quantum precision measurements [1]. Ensuring the spatial uniformity of the applied static magnetic field is crucial for these applications. The main methods for producing a uniform static magnetic field are to use Helmholtz coils, solenoids, and four-conductor assembly [2-5]. The four-conductor assembly has been successfully used in cesium beam primary frequency standards or fountain clocks [6, 7].

The conventional four-conductor assembly consists of four straight conductors carrying equal currents. By properly setting the spacings between the four conductors, a uniform transverse static magnetic field along the central longitudinal axis of the structure can be obtained. Increasing the conductor length is typically required to improve field spatial uniformity or enlarge uniform region. This is feasible for laboratory environments, but unsuitable for engineering applications such as compact optically pumped cesium beam atomic clocks.

In this paper, we introduce an improved four-conductor structure by modifying the shape of four conductors. In order to verify the potential of this modified structure, we take its application in cesium beam clocks as an illustration example. As we will demonstrate below, the purely curved structure is superior to the conventional straight structure, and the hybrid structure combining straight sections and curved sections is more flexible in manipulating the field distribution. We also detailed how to obtain the expected field distribution by tuning the two additional degrees of freedom in addition to the conductor spacings in the conventional straight four-conductor assembly.

## II. STRUCTURE AND RESULTS

Fig. 1 (a) depicts the structure diagram of the modified four-conductor assembly proposed in this work. Each conductor consists of one centered straight section with a length of  $l$  and a pair of curved sections with a radius of  $r$  located at the two ends of the conductor. The cases of  $r = \infty$  and  $l = 0$  mm correspond to the conventional straight four-conductor and the purely curved structure, respectively. The direction of current  $I$  on each conductor is shown as the red arrow. The distance between two conductors having the same current direction is  $w$ , and the distance between the upper and lower conductors is  $h$ . The geometric parameters are detailed in Figs. 1(b) and 1(c).

The green color in Fig. 1 represents the region of interest. For cesium beam clock application, it represents the atomic beam region. What we need is to produce a transverse magnetic field  $B_y$  as uniform as possible in the beam region to separate the  $|3, 0\rangle \leftrightarrow |4, 0\rangle$  clock transition from other static magnetic sensitive transition lines. The direction of  $B_y$  is the direction of the quantization axis. We will show that the static magnetic field distribution for a given conductor length  $L$  can be significantly modified by changing the values of additional structural parameters,  $l$  and  $r$ , in addition to  $w$  and  $h$ .

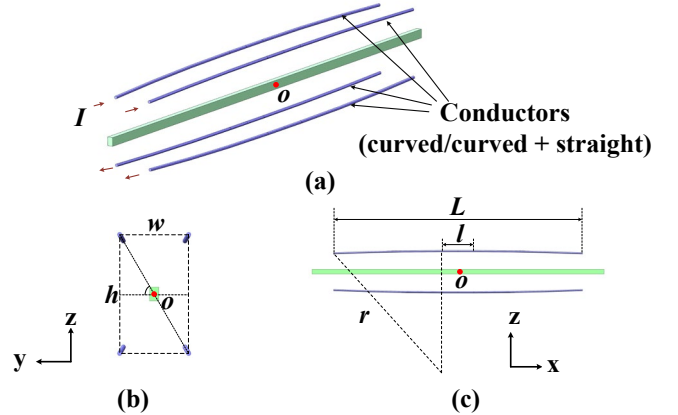


Fig. 1. Structure of the modified four-conductor assembly.

We begin our discussion from the straight four-conductor assembly. The transverse component  $B_y(x, 0, 0)$  of the static magnetic field along the central x-axis can be analytically calculated according to the Biot-Savart law, which reads:

$$B_y(x, 0, 0) = \frac{\mu_0 I h}{2\pi D^2} \left[ \frac{\frac{L}{2} - x}{\sqrt{D^2 + \left(\frac{L}{2} - x\right)^2}} + \frac{\frac{L}{2} + x}{\sqrt{D^2 + \left(\frac{L}{2} + x\right)^2}} \right] \quad (1)$$

where  $\mu_0$  is the magnetic permeability in vacuum, the field point  $x \in (-L/2, L/2)$  and  $4D^2 = w^2 + h^2$ .

The non-uniformity of the static magnetic field is defined as

$$\eta = \frac{B_y(x, 0, 0) - B_y(0, 0, 0)}{B_y(0, 0, 0)} \quad (2)$$

where  $B_y(0, 0, 0) = \frac{\mu_0 I h}{\pi D^2} \frac{L}{\sqrt{4D^2 + L^2}}$  is the reference magnetic field at the midpoint on the central x-axis.

For a compact optically pumped cesium beam atomic clock with E-plane bent Ramsey cavity (Fig. 2), the cesium atoms interact with the laser and are optically pumped to the clock state in the first optical region. Then the clock state atoms pass through a Ramsey cavity. Finally, the clock signal is extracted in the second optical region. The traveling direction of the atomic beam is perpendicular to the direction of microwave and static magnetic fields. The separation of the two spatially separated microwave magnetic fields is about 120 mm. Considering the free drifting region and both optical regions, we set  $L = 318$  mm and  $w = 30$  mm. As often done in the straight four-conductor assembly, we let  $h = \sqrt{3}w$  [5].

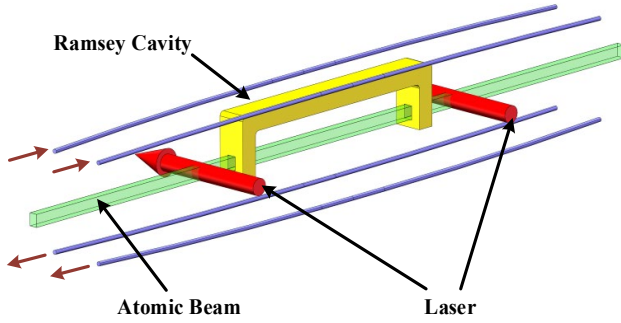


Fig. 2. Schematic view of the optically pumped cesium beam atomic clock. A highly collimated cesium atomic beam ejected from the cesium source (not shown) successively interacts with pumping light, microwave magnetic field, static magnetic field and probe light. The static magnetic field covers all the oscillating field-atom interaction regions to minimize dark state effects.

We first consider the case of  $l = 0$  mm, where the bending characteristic of the four-conductor assembly can be described by radius  $r$ . Fig. 3 shows the simulated non-uniformity of  $B_y$  under different  $r$ . It is clear that the static magnetic field distribution is significantly modified while varying the radius  $r$ . The edge field increases with the decrease in the radius  $r$ , thus reducing the field falling of the finite length straight conductor. When  $r$  becomes large enough, the field distribution produced by the purely curved four-conductor assembly gradually closes to that produced by the conventional straight four-conductor assembly (dash line in Fig. 3). The comparison of the field non-uniformity created by curved and straight structures shows that

it is possible to produce a highly uniform static magnetic field distribution by properly designing the shape of the finite length conductors. We obtained an optimized non-uniformity of below  $10^{-3}$  over a range of 140 mm covering the whole Ramsey cavity region, as shown in Fig. 4. This helps to reduce the center offset between the Ramsey fringe and the Rabi pedestal.

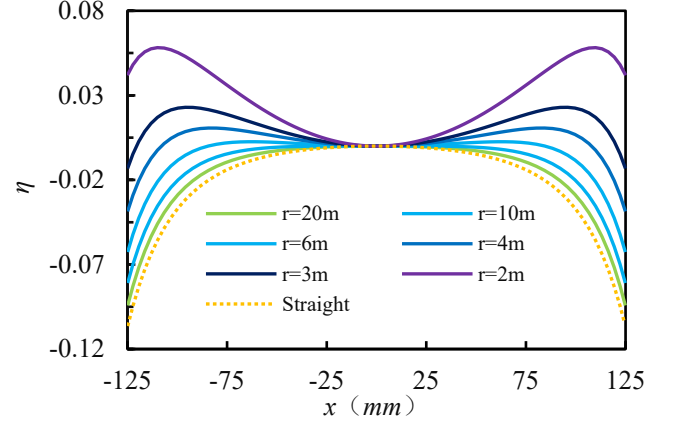


Fig. 3. Non-uniformity of the transverse component  $B_y(x, 0, 0)$  of the static magnetic field along the central x-axis while  $w$  and  $h$  remain unchanged. For conventional straight four-conductor structure, the non-uniformity of 1% only corresponds to a range of about 15 mm around the center.

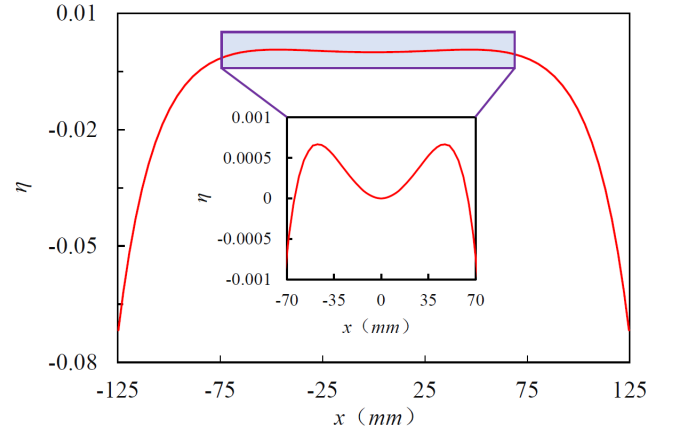


Fig. 4. Non-uniformity of  $B_y(x, 0, 0)$  when  $r = 7.65$  m.

It should be noted that the atomic pumping efficiency is directly related to the performance of the optically pumped cesium beam atomic clock. The zero or low static magnetic field in the optical pumping region may lead to a reduction in the population difference  $\Delta n$  between the two ground states, thus worsening the clock signal. This can be well interpreted by the Zeeman coherence effects. Increasing the static magnetic field intensity in the optical region has been proved to be an effective means to cancel the coherence [8].

We find that the curved four-conductor assembly with a small-radius can produce a stronger static magnetic field in the optical region than in the cavity region, which is very beneficial for increasing the pumping efficiency. However, the enhancement of the field intensity in the optical region results in a slightly weaker field distribution around the central region.

To compensate for this effect, the conductors are bent only at their ends, so that each conductor actually consists of one straight section and two curved sections. As a demonstration example, Fig. 5 shows the non-uniformity of the static magnetic field created by a hybrid four-conductor assembly. It has been shown that the effect of both the straight section and the curved sections is important on the field spatial distribution. Note that there is no reason to keep  $h = \sqrt{3}w$  strictly. By fully optimizing all degrees of freedom, it is possible to obtain more suitable parameters for an optically pumped cesium beam clock.

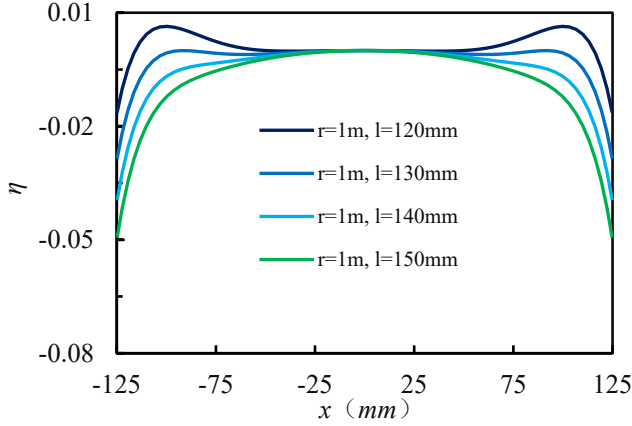


Fig. 5. Non-uniformity of  $B_y(x, 0, 0)$  created by hybrid structure when  $r = 1$  m.

Fig. 6 shows the simulated magnetic flux density in the yoz cross-section ( $1 \text{ mm} \times 4 \text{ mm}$ ) of the cesium beam at  $x = 0$  mm, where all four conductors carry a 1 A current. The illustration exhibits a uniform static magnetic field distribution within the region of the atomic beam.

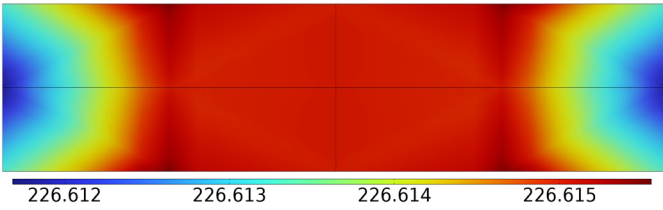


Fig. 6. Static magnetic field distribution on the cross section of the atomic cesium beam at  $x = 0$  mm. The unit is mG.

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## III. DISCUSSION AND CONCLUSION

Obtaining a highly uniform static magnetic field is critical in atomic precision spectroscopy experiments. We present a curved four-conductor assembly, derived from the modification of conductor shape, that exhibits enhanced uniformity of the static magnetic field compared to the conventional straight four-conductor assembly. We demonstrated how to construct the expected spatial distribution of the magnetic field by manipulating the additional degrees of freedom to fulfill the specific application requirements.